APPENDIX D – DIRECT CURRENT RESISTIVITY AND INDUCED POLRIZATION (DCIP) THEORY

INTRODUCTION

The resistivity is among the most variable of all geophysical parameters, with a range exceeding 10^6. Because most minerals are fundamentally insulators, with the exception of massive accumulations of metallic and submetallic ores (electronic conductors) which are rare occurrences, the resistivity of rocks depends primarily on their porosity, permeability and particularly the salinity of fluids contained (ionic conduction), according to Archie's Law. In contrast, the chargeability responds to the presence of polarizeable minerals (metals, submetallic sulphides and oxides, and graphite), in amounts as minute as parts per hundred. Both the quantity of individual chargeable grains present, and their distribution within subsurface current flow paths are significant in controlling the level of response. The relationship of chargeability to metallic content is straightforward, while the influence of mineral distribution can be understood in geologic terms by considering two similar, hypothetical volumes of rock in which fractures constitute the primary current flow paths. In one, sulphides occur predominantly along fracture surfaces. In the second, the same volume percent of sulphides are disseminated throughout the rock. The second example will, in general, have significantly lower intrinsic chargeability.

More detailed descriptions on the theory and application of the IP/Resistivity method can be found in Van Blaricom (1992) and Telford et al. (1976).

HALVERSON-WAIT CHARGEABILITY

The Titan-24 DCIP chargeability decays are described using the Halverson-Wait spectral model (Halverson et al., 1981), which is not well known, but is similar to the Cole-Cole model proposed by Pelton et al. (1978) which is a simple relaxation model that fits complex (frequency-dependant) resistivity results.

The time domain chargeability, originally proposed by Siegel (1959), is defined (Telford et al., 1976) as:

$$M = \frac{1}{V_C} \int_{t1}^{t2} V(t)\,dt$$

Where $\boldsymbol{V(t)}$ is the residual or secondary voltage at a time $\boldsymbol{t}$, that is decaying after the current is cut off, between time t_1 and t_2, with the steady voltage V_C during the current flow interval. The ratio $V(t)/V_C$ is expressed in millivolts per volt.

In the frequency domain, the "frequency effect" is defined as:

$$fe = (\rho_{DC} - \rho_{AC}) / \rho_{AC}$$

Where ρ_{DC} and ρ_{AC} are apparent resistivities measured at d.c. and "very high" frequency, usually in the 0.1 to 10 Hz range. The Cole-Cole model for the chargeability $\boldsymbol{m}$, as defined by Pelton et al. (1978) is given by the following:

$$Z(\omega) = R_0\left[1 - m\left(1 - \frac{1}{1 + (i\omega\tau)^C}\right)\right]$$

Where $Z(\omega)$ is the complex impedance, R_0 is the DC resistivity, $\boldsymbol{m}$ is the chargeability in volts per volt, ω is the angular frequency in Hz, τ is the time constant in seconds, and $\boldsymbol{c}$ is the frequency dependence (unitless). The latter two physical properties describe the shape of the decay curve in time domain or the phase spectrum in frequency domain, and commonly range between 0.01s to +100s and 0.1 to +0.5, respectively (Johnson, 1984).

The Halverson-Wait model was proposed by Halverson et al. (1981) as an extension to the Wait (1959) model of the impedance of "volume loading" of spheres, given by:

$$Z(\omega)=\frac{\rho}{G}\left[1-3v\left(1-\frac{3\delta}{1+2\delta}\right)\right]$$

Where G is a geometric factor, ρ is the resistivity of the media, v is the volume loading (the volume fraction of chargeable "spheres"), δ is the sphere surface impedance. The Wait model was designed to provide an explanation of the differences in the shape of decay curves from different polarizeable targets, but does not describe very well the physical attributes of the rocks.

The Halverson-Wait model expands the Wait coated sphere IP model to include a new formulation of the sulphide-rock interface impedance, based on field studies and laboratory tests on samples. It is closely correlated to the Pelton et al. (1978) Cole-Cole model and is given by:

$$Z(\omega)=\frac{\rho}{G}\left[1-3v\left(1-\frac{3/2}{1+r[i\omega]^{\kappa}}\right)\right]$$

Where r is the sphere radius and is equivalent to τ - the Cole-Cole time constant ($r = \tau^{K}$). The v volume loading compares well to m – the Cole-Cole chargeability (see equation below) – and the exponent k is equal to c - the Cole-Cole frequency dependence (Halverson et al., 1983). For sulphide systems, the r-factor reflects the size or interconnection of the sulphide grains and the k-factor reflects the electrical characteristics of the sulphide surfaces. An example of time domain Halverson-Wait model responses is shown in Figure J.1.

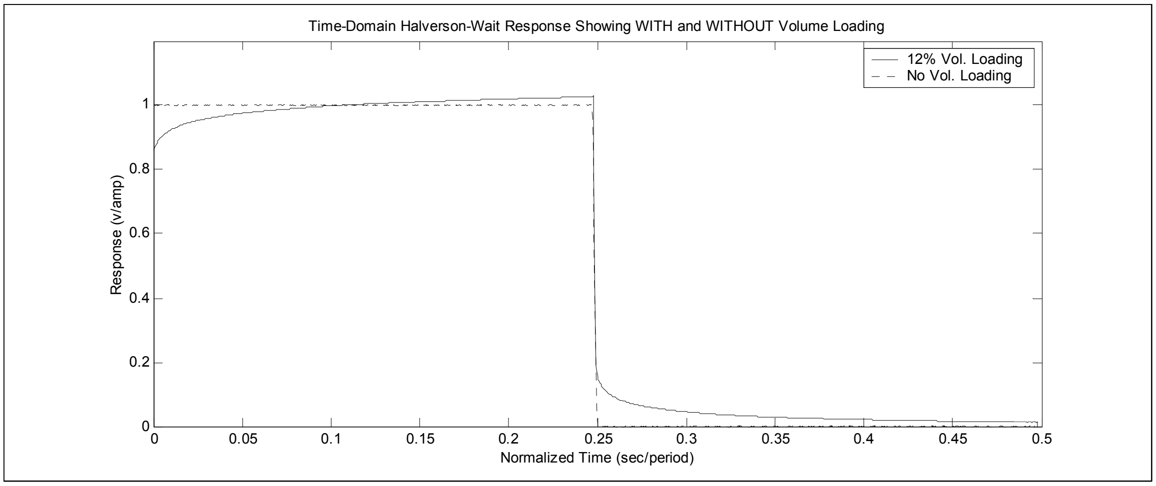

Figure J.1: Polarizeable versus Non-Polarizeable TDIP Response using Halverson-Wait Model

In practice the Titan chargeability decays are fit to a Halverson-Wait model. In order to solve for the volume loading v, the r-factor and k-factor are set to the standard (typical) Halverson-Wait values of 1.0 and 0.2, respectively. In the Halverson-Wait model the theoretical PFE (for infinite bandwidth), which equates to the theoretical chargeability in the Cole-Cole equation, is thereby defined by the volume loading:

$$PFE_0 = m_0 = \frac{9v}{(2+3v)}$$

and m is output in units of milliradians.

INVERSION THEORY

An excellent overview and introduction to both the philosophy and use of inversions in geophysics is available on the University of British Columbia (UBC) website (http://www.geop.ubc.ca/ubcgif/; Oldenburg et al., 1998).

Several points, detailed on the website, are crucial to understanding the Titan-24 approach to exploration:

- Inversion is a powerful 'tool', not a 'solution'.
- Inversion is not normally "unique". Given noisy and incomplete data of inherently limited resolution there are usually an 'infinite' range of models that 'fit' the data equally well. Recognition of this inherent non-uniqueness is why inversion must be viewed as a tool rather than a solution. Understanding and exploration of this non-uniqueness is an important part of the interpretive process.
- Inversion finds a model that 'fits' the data. The precise definition of 'fit' can be critical in the actual model that is found.
- The inversion depends on the data, and the data errors. The importance of the data errors is often overlooked.
- Inversion depends on a "model norm" – the mathematical definition of which model the inversion should try to find. This definition is almost as important as the actual data in determining the final inversion model.

Mathematically, inversion is the process of minimizing a function. The choice of which function to minimize ultimately defines the inversion model. Schematically, this function might be expressed:

$$\phi = \phi_d + \beta\,\phi_m = (\text{misfit}) + \beta\,(\text{model norm})$$

$$0 < \beta < \infty \quad \text{is a constant}$$

This defines a function to be minimized that consists of some function that minimizes the data misfit, combined with some function that finds a "smooth" model. Beta represents a relative weighting between fitting the data and smoothing the model.

Clearly, the data misfit function must be defined in more detail. One approach might be:

$$\phi_d = \sum_{i=1}^{N} \left(\frac{F_i[m] - d_i^{\,obs}}{\varepsilon_i} \right)^2$$

This function defines the data misfit as the sum of the individual misfits squared, normalized by the errors associated with each data point. It is a very common, and stable, definition of the data misfit.

An important point not made on the UBC website is that the errors depend on many factors. The most common measure of data errors is simply the repeatability of the voltage and current measurements in the field. This may be misleading as there are also "errors" associated with electrode positioning, geologic complexity (2D vs 3D, but also coupling of shallow and deeper structure), and errors in the numerical calculation of model responses and inversion.

Another point not sufficiently detailed on the UBC site is the importance of not overestimating the data errors and fitting the data as closely as possible. Most geophysical techniques, but particularly electrical techniques, have large responses to shallow structure. This is expressed as "pant legs" in DC/IP, or "statics" in MT. The response to deep structure is generally a very subtle component of the data, compared to the sensitivity to shallow structure. Without excellent data, and an excellent match between the data and model response, the deep structure will not be imaged to the degree necessary for commercial exploration.

The model misfit function must also be defined in more detail. One of the most flexible definitions is the one used by UBC:

$$\phi_m(m,m_0) = \alpha_s \int_{vol} (m-m_0)^2 dv + \alpha_x \int_{vol} \left(\frac{\partial(m-m_0)}{\partial x} \right)^2 dv + \alpha_z \int_{vol} \left(\frac{\partial(m-m_0)}{\partial z} \right)^2 dv$$

In this definition there are three components to the "model norm" (or "smoothness" constraint, or "regularization"), each of which contains an α constant (α_S, α_X, α_Z) that are commonly referred to as "alpha parameters". The first component is simply an overall difference between the model and a "target" model, the second component is a horizontal smoothness, and the third component is a vertical smoothness. The three "alpha" parameters (α_S, α_X, α_Z) represent a relative weighting of each component. A fourth variable, m_0, refers to the starting or reference model – either a half-space or geophysical constraint – that also has a profound influence on the model-misfit.

The UBC website provides an excellent example of the importance of selecting an appropriate "model norm", reproduced in Figure J.2

In this example the expected response of the top figure was computed. These 'data' were then inverted six times, using different "model norms" (α_S, α_X, α_Z, m_0). The lower six figures show the range of valid inversion models that can be produced. Note that six of these models are essentially mathematically equivalent, they all "fit" the data.

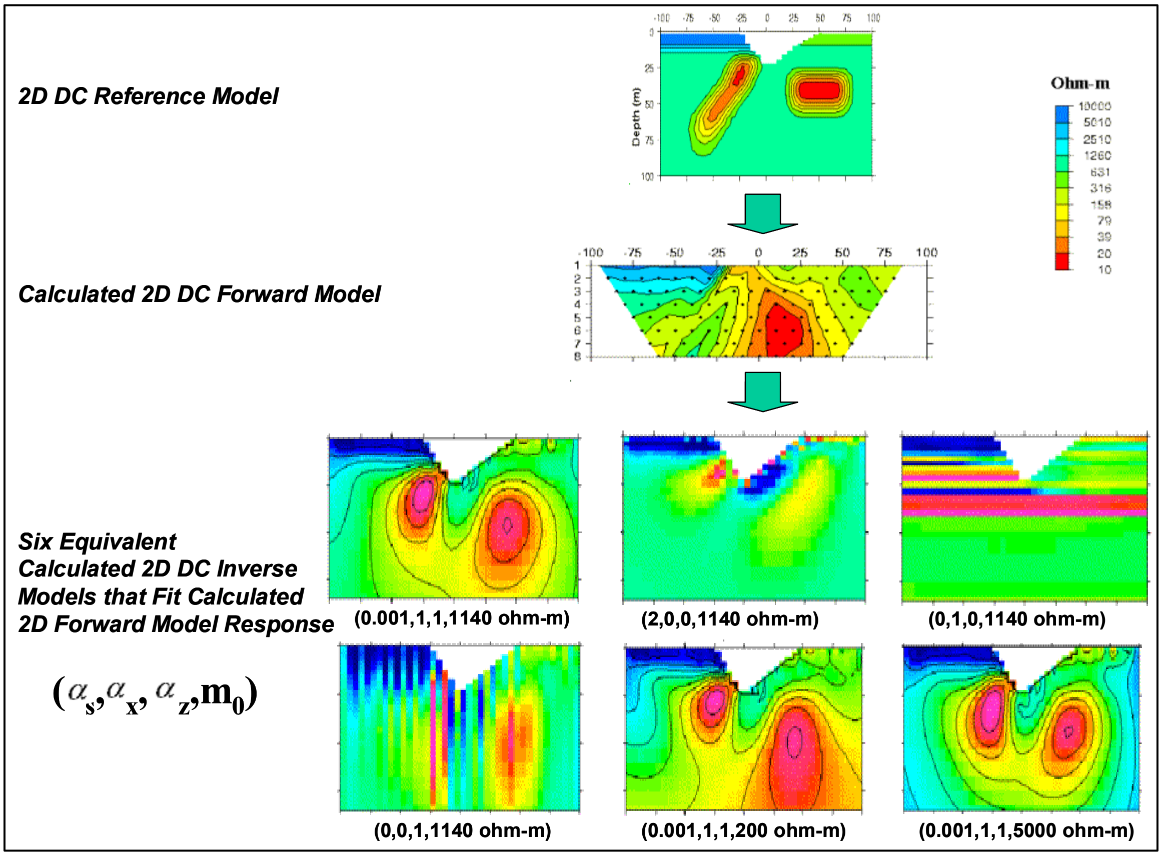

Figure J.2: Effects of Model Norm and Starting Model on Inversion Results (modified after Oldenburg, et al.,1998).

An important philosophy, driving much of the academic communities approach to inversion for the last two decades, is that the "best" model is the "smoothest" model consistent with the data. There are good reasons for taking this approach. However, from an exploration viewpoint this philosophy can be rephrased to "find the model with the least exploration value" – perhaps not reflecting the real goal of an exploration program.

Recently, several groups have taken major steps towards developing inversion approaches more tuned to exploration needs. Instead of using "smooth" model norms, they are being replaced with "focused (minimum transition zone) inversion, or smoothing to a geologic "target" model.

For exploration smoothing to a geologic target model makes sense. It requires good geologic control, and some understanding of the rock physical properties. There are three drawbacks to the geologic target approach:

- The geologic information is incomplete or inaccurate.
- Physical property data are incomplete.
- It is difficult to determine whether the geophysical data support the geologic model, or simply provide no information.

The most sensible approach is to combine smooth model inversion with geologic target inversion. For now, we are focusing on providing inversions using both approaches. It is up to the project geologist and geophysicist to review these inversions and develop a final interpretation.